

VII. STANDING COMMITTEES

A. Academic and Student Affairs Committee

Student Presentation

Department of Mathematics
Putnam Competition

VII. STANDING COMMITTEES

A. Academic and Student Affairs Committee

Student Presentation (continued p. 2)

University Week - April 1, 2010

UW student wins mathematics competition, named Putnam Fellow

By Hannah Hickey
News and Information

Last month stadiums reverberated as students on the UW's basketball team made it to the Sweet Sixteen round of the National Collegiate Athletic Association tournament. Meanwhile, over spring break another UW undergraduate quietly claimed the top prize in U.S. collegiate mathematics. William Johnson, who is majoring in mathematics and computer science, was named a Putnam Fellow, placing among the top five out of more than 4,000 students who competed this year.

While this competition allows no spectators, winning the Putnam is no less a feat than bringing home the NCAA title -- especially when it's an upset.

"Just as Duke, Kansas, and Kentucky always seem to dominate in basketball, the Putnam Fellowships have been 'owned' by Harvard, MIT and Cal Tech. It's great to have a Husky join them," wrote President Mark Emmert. "Our math department is truly remarkable in working with our students, and a real point of pride for us."



William Johnson, Putnam Fellow

The William Lowell Putnam Mathematical Competition is held each December by the Mathematical Association of America. The competition began in 1938, and is open to undergraduate students in the United States and Canada. The UW team had a strong finish last year (see our story [here](#)), but this is the first time a UW student has been named a Putnam Fellow.

Johnson grew up in the Seattle area and attended Kenmore's Inglemoor High School. Last year he placed sixth overall in the Putnam, just two points away from being among the winners. He was recently named the UW's Junior Medalist for earning the highest overall academic record for his class. This year he wins \$2,500 and the honor of being named a Fellow, a distinction that will follow him through his career.

The contest is the most prestigious in U.S. mathematics circles. When mathematician Jonathan Nash, subject of the book and movie *A Beautiful Mind*, would first meet other mathematicians he reportedly would ask whether they had taken the Putnam and how they placed.

"This is huge," said Selim Tuncel, chair of the mathematics department, noting that the list of previous winners includes many of the top names in the field. "Will's achievement is a combination of amazing talent and excellent mentoring on the part of my colleagues."

The UW team has been coached for the past two years by Ioana Dumitriu, a UW assistant professor of mathematics who in 1996 was the first woman to be named a Putnam Fellow, and Julia Pevtsova, also a UW assistant professor of mathematics, who was a silver medalist in the International Mathematical Olympiad.

A-6/205-10
5/13/10

VII. STANDING COMMITTEES

A. Academic and Student Affairs Committee

Student Presentation (continued p. 3)

"I am very, very proud of Will," Dumitriu said. "For me, it's a matter of huge pride to have a Putnam Fellow that I helped train."

The coaches emphasize that they cannot take credit for Johnson's performance.

"No amount of coaching could get him there unless he was willing to put in the work and unless he had this special talent to begin with," Dumitriu said. "It's kind of like athletes. There's a tremendous amount of work that has to be put in, on top of a very good natural ability."

And, like athletes, a winning score requires focus and stamina on game day. The Putnam is a six-hour contest. Competitors are given one set of problems in the morning and another in the afternoon. They must submit fully written-out proofs to get full credit. Of a possible 120 points, the average competitor scores 1 or 2. (Johnson scored around 100.)

During fall quarter Dumitriu and Pevtsova co-taught [Math 480a](#), The Art of Problem Solving, which prepares students to write the Putnam (students in the course are not required to enter the contest). Pevtsova and Dumitriu also hosted weekly evening Putnam practice sessions that were attended by about 12 regulars and as many as 30 students.

This year 19 UW students wrote the Putnam. Four others placed in the top 500: Yisong Song, a freshman in pre-sciences, Steven Rutherford, a freshman in computer engineering and Keyun Tong, a senior in computer science and Nate Bottman, a senior in Russian and mathematics, who both placed in the top 500 last year.

In addition to the coaching, Johnson credits his success to his religious beliefs, parents who encouraged an interest in mathematics from an early age, good math teachers, and two years of participation in the Mathematical Olympiad Summer Program in Nebraska.

None of Johnson's teachers was surprised to learn of his win.

"Will will be famous. I don't know what he will choose to do. It doesn't matter. He will add originality and depth to anything he tackles," wrote Jim Morrow, a UW professor of mathematics and one of the teachers Johnson singled out as an influence.

And while Johnson excels in theorems and proofs, he also shows interest in applied problems. About a year ago Johnson approached Richard Ladner, UW professor of computer science and engineering, to help with his mobile accessibility research because he wanted to work on a project that could have a positive impact on people.

On his own initiative Johnson created a program that uses the vibration of an Android phone to transmit Braille through the touch screen. Johnson's tool, dubbed V-Braille, has been tested by members of the local deaf-blind community.

"I have shown his V-Braille to colleagues around the country who have told me that V-Braille is 'brilliant,' 'stunning,' and 'you should patent it,'" Ladner writes. He says he has seldom met a student "who has such prodigious talent, works hard, and is so creative."

The other four Putnam Fellows this year hailed from Harvard, Yale and MIT. Though Johnson probably could have had his pick of these, he chose to attend the UW.

VII. STANDING COMMITTEES

A. Academic and Student Affairs Committee

Student Presentation (continued p. 4)

"I liked the campus, and I like the state of Washington, where I grew up," said Johnson, whose tuition was paid through the Washington Scholars program.

Johnson has at least one more year of study at the UW. After graduating he is considering working in computer programming or pursuing a graduate degree in mathematics.

VII. STANDING COMMITTEES

A. Academic and Student Affairs Committee

Student Presentation (continued p. 5)

The Seattle Times

Editorial - Friday, April 9, 2010

Brains, talent, hard work add up to a rare academic honor

WILLIAM Johnson, a University of Washington mathematics and computer-science major, is the pride of the university, his hometown Kenmore and Inglemoor High School. He is the absolute, undisguised envy of a lot of very smart people around the globe.

Johnson will be known forever and all time as a Putnam Fellow, a winner of The William Lowell Putnam Mathematical Competition, hosted every December by The Mathematical Association of America.

This is an extraordinary achievement with the capacity to delight the rest of us who are puzzled by Venn diagrams and subject to arithmetical second-guessing by the Internal Revenue Service.

He placed among the top five out of 4,000 students who dared test themselves against the nation's finest collegiate academic all-stars. The highest possible score is 120 points, and most competitors earn fewer than 10. Johnson is thought to have scored in the 100-point range.

The fearsome essence of the challenge is summarized on the Putnam Web site: "The examination will be constructed to test originality as well as technical competence." Beyond adroitly recalling what they have been taught, the best and brightest must apply what they have learned.

Johnson is the first UW student to be named a Putnam Fellow, an academic appellation that will follow him through his professional career. Not unlike college freshmen swapping SAT scores in the dorm, any random group of mathematicians will sort itself by Putnam scores.

Johnson's tenacious brain power and academic strengths were groomed for glory by two faculty stars, Ioana Dumitriu, an assistant professor of mathematics, who was the first woman named a Putnam Fellow, and Julia Pevtsova, another assistant professor, who was a silver medalist in the International Mathematical Olympiad.

Johnson's family, university, faculty mentors, and indeed his community and state can all take enormous pride in his hard work and prodigious capabilities. This is a rare achievement to be celebrated.

Here is a sample problem to get a sense of the competition:

Players 1, 2, 3, n are seated around a table and each has a single penny. Player 1 passes a penny to Player 2, who then passes two pennies to Player 3. Player 3 then passes one penny to Player 4, who passes two pennies to Player 5, and so on, players alternately passing one penny or two to the next player who still has some pennies. A player who runs out of pennies drops out of the game and leaves the table. Find an infinite set of numbers n for which some player ends up with all n pennies.

VII. STANDING COMMITTEES

A. Academic and Student Affairs Committee

Student Presentation (continued p. 6)

Competing Students

Putnam Fellow William Johnson

William grew up in Kenmore, WA, and went to Kenmore and Shelton View Elementary Schools, Kenmore Junior High, and Inglemoor High School. In high school he participated in the American Mathematics Competitions and did well enough to qualify for the USA Math Olympiad, and also to go to the Math Olympiad Summer Program in Nebraska. He is currently in his 3rd year of college and majoring in Computer Science major and, as of January 2010, Mathematics.

Will participated in the Putnam Competition in his 2nd and 3rd years of college. In his 2nd year, he placed in 6th place, two points short of being a Putnam Fellow. This year he did well enough to be in the top five, making him a Putnam Fellow, the first one from the UW. As an undergraduate, Will has done research in computer science, both in theory and in applications to improve accessibility for blind and deaf-blind people. This summer he plans on doing research at UW with the Department of Mathematics.

Other Students Scoring Above 20

Nate Bottman (senior)

Steve Rutherford (junior)

Yisong Song (freshman)

Igor Tolkov (senior)

Keyun Tong (junior)

Faculty

Selim Tuncel

Selim Tuncel was born in Istanbul in 1957. After completing his secondary education at Robert College, he attended the University of Sussex and the University of Warwick in England, receiving his BSc with First Class Honors in 1978 and PhD in 1982. Following postdoctoral appointments at the University of Washington, the Mathematical Sciences Research Institute in Berkeley and the Institute for Advanced Study, he joined the UW Math Department as an assistant professor in 1987. He was promoted to professor in 1993 and became department chair in 2002. He is married to Karin Bornfeldt, also a UW faculty member, and they have two sons, Miles and Dylan, aged 11 and 6.

VII. STANDING COMMITTEES

A. Academic and Student Affairs Committee

Student Presentation (continued p. 7)

Faculty (continued)

Ioana Dumitriu

Ioana's experience with math competitions started in elementary school and continued through high school and college; she usually ranked high in the Romanian National math championships. As an undergraduate at New York University, Ioana participated in the Putnam competition four times, winning it in her second attempt. After getting a PhD in Mathematics from Massachusetts Institute of Technology, Ioana accepted a postdoctoral Miller Fellowship at University of California at Berkeley. While at Berkeley, she was asked to help coach their Putnam team. She enjoyed her new-found calling as a coach so much that, once she became an Assistant Professor at University of Washington, she jumped at the opportunity to train UW's Putnam team. With the arrival of a like-minded colleague, Julia Pevtsova, who joined forces to create a Putnam tradition at UW, the future is looking bright.

In her research life, Ioana is the recipient of many prizes, including a Honorable Mention in the Householder Competition, the Leslie Fox Numerical Analysis Prize, and an NSF CAREER Award in 2009. She is currently supervising two graduate students.

Julia Pevtsova

Julia's competitive experience dates back to her high school days when she participated three times in the Soviet National Math Olympiad, coming in 3rd in the country in her senior year. Following that, she became part of the Russian team for the International Math Olympiad earning a Silver medal. Once she became a student at the Saint Petersburg State University, she combined her studies and research with coaching gifted kids in math. The coaching part was put on hold upon moving to the US and getting a PhD in mathematics at Northwestern University and then holding temporary positions at the Institute of Advanced Study in Princeton and the University of Oregon. In the Fall of 2008, Julia became an Assistant Professor at UW and formed a partnership with Ioana Dumitriu with the goal of creating a Putnam tradition at UW.

In addition to working with the UW's most mathematically talented students, Julia teaches at the Summer Institute of Mathematics at the University of Washington, a math summer camp for high school students, and organizes Math enrichment programs for elementary and middle schools students in Seattle. Julia's research has been supported by the National Science foundation since 2005; in particular, she was awarded a CAREER grant by the NSF in 2010.

The Putnam Mathematical Competition: Facts and Statistics

Ioana Dumitriu and Julia Pevtsova

May 13, 2010

A-6.1/205-10
5/13/10

Introduction

- The William Lowell Putnam Mathematical Competition is the most prestigious undergraduate mathematical competition in the US and Canada, both at individual and team level.
- It has been held annually since 1938, making it the oldest intercollegiate competition of its kind.
- The Putnam competition has a very rich history and has been extremely successful at identifying extraordinary mathematical talent.

Description

Format:

- ◇ It is held on the first Saturday in December;
- ◇ It consists of two sessions, each lasting three hours;
- ◇ There are six problems per session, which the contestants must prove and write solutions for.

Winners and winning teams:

- The Top Five individual scorers are designated as the winners and declared "Putnam Fellows";
- The top 60-80 individual scores receive an Honorable mention or various other distinctions (e.g., Top Ten);
- Each participating college or university has a pre-designated three-member team, ranked according to the sum of the members' individual ranks; the Top Five teams are declared winners.

A-6.1/205-10
5/13/10

A Putnam's "WHO'S WHO"

Among the laureates of the Putnam Competition we count

- Four Nobel Prize winners in Physics:
 - Richard Feynman and Kenneth Wilson (Putnam Fellows);
 - Steven Weinberg and Murray Gell-Mann (Honorable Mentions).
- One Nobel Prize winner in Economics: John Nash (Top Ten);
- Five Fields Medal winners (The equivalent of the Nobel Prize for Mathematics):
 - John Milnor, David Mumford, and Daniel Quillen (Putnam Fellows);
 - Paul Cohen (Top Ten) and John G. Thompson (Honorable Mention);
- Four presidents of the American Mathematical Society:
 - Irving Kaplansky, Andrew Gleason, Felix Browder (Putnam Fellows);
 - Ron Graham (Honorable Mention);
- Eric Lander, principal investigator in the Human Genome Project (Top Ten);
- To date, at least fourteen Putnam Fellows have been elected to the National Academy of Sciences.

- The number of participants, each year, is in the 4000s;
- The median score is between 0-2 (out of a possible 120);
- The percentage of 0 scores is between 40-60%;
- Making the Top 500 is a very respectable achievement and requires solving at least two problems.

Recent Statistics, II

- The winning teams in the 00's are: Harvard, MIT (9 times each); Duke, Princeton (7 times each); Stanford (5 times each); Waterloo (4 times); Caltech (3 times); UC Berkeley, Toronto (2 times each); U. Chicago, Harvey Mudd (1 time each).

- Number of undergraduates from US public universities that have been named Putnam Fellows in the 90's: **3**
 - Jordan Lampe, UC Berkeley;
 - Xi Chen, Missouri-Rolla;
 - Ovidiu Savin, U Pittsburgh;

- Number of undergraduates from US public universities that have been named Putnam Fellows in the '00s: **2**
 - Jan Siwanowicz, CUNY;
 - **William Johnson**, U of Washington.

The Putnam Competition from 1938-2009*

Joseph A. Gallian

1. INTRODUCTION. The William Lowell Putnam Competition is held annually for the top undergraduate mathematics students in the United States and Canada. The first Putnam competition took place in 1938, but its genesis was a math competition held in 1933 between ten Harvard students and ten students from the United States Military Academy at West Point [2]. That competition was sponsored by Elizabeth Lowell Putnam in honor of her late husband William Lowell Putnam, who was a member of the Harvard class of 1882. That competition went so well that plans were made to have an annual competition in which all interested institutions could participate. This came about in 1938, when the first official Putnam competition was sponsored by the Mathematical Association of America. The examination was prepared and graded by members of the Harvard mathematics department and Harvard students were excluded the first year. There were both individual and team competitions. The questions were drawn from calculus, the theory of equations, differential equations, and geometry. (The problems are included at the end of this article.) Prizes in the first few years were \$500, \$300, and \$200 for the top three teams and \$50 each for the top five ranking individuals, who were designated as Putnam Fellows. By the year 1997 the prizes for the top five teams were \$25,000, \$20,000, \$15,000, \$10,000, and \$5,000, while Putnam Fellows received \$2,500 each. Moreover, each year one Putnam Fellow receives the William Lowell Putnam Fellowship for graduate study at Harvard.

The first competition had 163 individuals and 42 teams. The number of participants exceeded 1,000 for the first time in 1961, when 1,094 individuals and 165 teams took part. In 2009 there were 4036 students representing 546 institutions and 439 teams. All three of these totals were record highs. The number of participants in the 2009 competition alone exceeds the total number of participants in the first eighteen competitions from 1938 through the spring of 1958. (The competitions were suspended from 1943-1945 because of World War II; in 1958 there were two competitions—one in the spring and one in the fall.) Coincidentally, in both 1980 and 1981 there were exactly 2,043 participants. Through 2009, there have been 118,868 participants. The 1946 contest, coming right after the war, had the lowest participation ever with just 67 contestants and 14 teams. Table 1 at the end of this article provides the list of the number of participants in each of the seventy competitions through 2009.

In the first twenty-two competitions the number of questions varied from eleven to fourteen, but beginning with the 23rd competition in 1962, the exams have consisted of a three-hour morning session and a three-hour afternoon session, each having six questions worth ten points apiece. Institutions entering teams must designate the three team members before the competition is held. The team score is the sum of the ranks of the three team members. Thus, a team whose members finish in twenty-first, forty-ninth, and one hundred and second places has a score of 172. The lower a team's score, the higher its ranking. This method of team scoring places great weight on the lowest scoring member of the team since there is much bunching at lower scores. For example, in 1988 a team member with a score of ten ranked 1496, but a team member with a score of nine ranked 1686. In 2006 a score of one point generated 1266.5 team points, whereas a score of zero on that exam resulted in 2501 team points. Thus, even a one point difference in an individual's score can mean over a thousand points more for the team.

The fact that the team members are designated in advance and the method of summing the ranks for team scoring causes some peculiar results on occasion. In 1959, for instance, Harvard had four Putnam Fellows but finished fourth in the team competition, and in 1966, 1970, 2005 and 2006 MIT

*This is an updated version of an article published in the American Mathematical Monthly [5] in 2004.

had three Putnam Fellows but did not win the competition. There have been sixteen competitions in which the winning institution did not have a Putnam Fellow.

One might wonder about the most difficult Putnam problems over the years. Using data from 1974-2009, the only problem for which no one in the top 200 received a positive score was A6 on the 1979 exam. In 1999 for both B4 and B5 only a single person in the top 200 received a positive score. In each instance the score was two. These three problems are reproduced in the Appendix II. In 2009 six people among the top 200 scorers had positive scores on B6.

2. TEAM PERFORMANCE. By a wide margin, Harvard has the best record in the Putnam competition. Through 2009, Harvard has won the team competition twenty-seven times, while its closest rival, Caltech, has won the team title nine times. MIT is in third place with six titles with three of these coming since 2003. Tied for fourth place with four team titles each are Washington University and the University of Toronto. All four of Toronto's team titles occurred in the first six years of the competition. Toronto might have won all of the first six competitions except for the fact that it chose to disqualify itself in 1939 and 1941 because the Toronto mathematics department had prepared the questions. Starting with the fifth competition the questions have been prepared by a committee selected from different schools rather than having the department of the winning team of the previous competition prepare them. This meant that the winner of the previous year would not have to disqualify itself. Curiously, the Harvard team did not place in the top five in the first six competitions, but it has placed in the top five in fifty-five of the seventy competitions held through 2009. During the first twenty competitions (1938-1959), the New York institutions Brooklyn College, Polytechnic Institute of Brooklyn, Columbia University, and City College of New York excelled in the team competition and in producing Putnam Fellows. Caltech's glory years were the six years 1971-1976 when they won the team competition five times. Excluding Harvard, only once has the same institution won three years in a row. That was Caltech in 1971-1973. Between 1976 and 1986 Washington University won the team title four times and placed second four times. During that period Wash U had only two Putnam Fellows. Beginning about 1990 Duke University started to recruit the nation's best high school math students with the same fervor that they recruit the best high school basketball players. Between 1990 and 2000 Duke became Harvard's top rival by winning three times and finishing second to Harvard twice. With these accomplishments together with its third place finish each year from 2001 to 2005, Duke's Putnam team has performed as well as its men's basketball team! (Through 2009 the men's basketball team finished first three times and second three times, with one other appearance in the final four.) After finishing in the top five twenty-four times and in second place nine times prior to 2006, Princeton won its first team title in 2006. The only state universities in the U. S. to win the team competition are Michigan State (three times), and the Universities of California at Davis (once) and at Berkeley (once). The highest place ever achieved by a liberal arts college was second by Oberlin College in 1972. That same year Swarthmore finished fourth. Harvard's longest winning streak was eight years (1985-1992), and its longest stretch without winning was fifteen years (1967-1981). The only tie for first place occurred in 1984 between the University of California at Davis and Washington University. Amazingly, in 1986, 1987, and 1990 every member of Harvard's team was a Putnam Fellow.

A complete list of the top five schools and top five individuals each year can be found at http://en.wikipedia.org/wiki/Putnam_competition. Table 3 lists every team that has placed fifth or higher in at least one competition along with the total number of Putnam Fellows from each of these institutions. The last four entries in the table list the institutions that have not placed in the top five in the team competition but have had at least two Putnam Fellows.

3. INDIVIDUAL ACCOLADES. As for producing Putnam Fellows, Harvard is again the overwhelming winner with ninety-eight versus MIT's second place fifty-one. On the other hand, between 2001 and 2009, MIT out did Harvard in Putnam Fellows twenty to eleven. Harvard has had four Putnam Fellows in the same competition on four occasions. Oddly, Harvard did not record its first Putnam Fellow until the sixth competition. Since then the longest period in which Harvard did not have a

Putnam Fellow is three years and that happened only once. Because of tie scores for fourth or fifth place, in fourteen competitions there have been six Putnam Fellows, while in 1959 a four-way tie for fifth place resulted in eight. Thirteen of the fifteen competitions in which there were more than five Putnam Fellows have occurred since 1970. Through 2009, there have been 269 individuals who have been Putnam Fellows for a total of 367, counting multiplicity. Only seven people—Don Coppersmith, Arthur Rubin, Bjorn Poonen, Ravi Vakil, Gabriel Carroll, Reid Barton, and Daniel Kane—have been Putnam Fellows four times. Eighteen people have been three-time winners: Andrew Gleason, Edward Kaplan, Donald J. Newman, James Herreshoff, Samuel Klein, Randall Dougherty, Eric Carlson, David Ash, Noam Elkies, David Moews, David Grabiner, Kiran Kedlaya, Lenny Ng, J. P. Grossman, Ciprian Manolescu, Aaron Pixton, Arnav Tripathy, and Yufei Zhao.¹ Zhao missed being a four time Fellow by one point in 2007. In Ash's fourth attempt at the Putnam in 1984 he finished tied for sixth, just two points short of being a Putnam Fellow again. It should be noted that some of the three-time winners only took the exam three times. Through 2009 there have been forty-two people who have been Putnam Fellows exactly twice. It appears that there have never been two members of the same immediate family who have been Putnam Fellows. The closest are brothers Doug and Irwin Jungreis. Doug finished in the top five in 1985 and 1986 and Irwin finished in the second five in 1980 and 1982. Dylan Thurston, son of Fields Medalist William Thurston, finished in the second five in 1993. The first certain occurrence of a woman finishing in the Honorable Mention or higher categories was in 1948. In the announcement in the *American Mathematical Monthly* [7] she is listed as "M. Djourup (Miss), Ursinus College." Because many participants use the initials of their first and middle names (e.g., R. P. Feynman) it is possible that Djourup is not the first woman to achieve Honorable Mention or better status. The first woman Putnam Fellow was Ioana Dumitriu from New York University in 1996; the second was Melanie Wood from Duke in 2002; the third was Ana Caraiani from Princeton in 2003 and 2004. Since the ages of participants are not noted, there is no way to know who the youngest and oldest people to win the competition were. Most likely the youngest is Arthur Rubin, who was a winner in 1970 at age 14. John Tillinghast, David Ash, Noam Elkies and Lenny Ng were Putnam Fellows at sixteen.² A potential oldest winner is Samuel Klein, who was born in 1934 and won the competitions in 1953, 1959, and 1960. As a group, the five winners of the 2003 competition have amassed the greatest number of Putnam Fellow designations ever: Gabriel Carroll, Reid Barton and Daniel Kane won four time, Ana Caraiani won twice, and Ralph Furmaniak won once.

Unlike the early years of the Putnam competition, in the past twenty-five years or so many of those who have done exceptionally well in the Putnam competition have participated as high-school students in problem solving summer training camps in the United States and elsewhere in preparation for the annual International Mathematics Olympiad (IMO). Many of the international students who represented their countries in the IMO have come to the United States for their undergraduate degrees. The consequence is that the winners of Putnam competitions now come from many countries. The 2006 Putnam competition illustrates this well. All five 2006 Putnam winners were IMO gold medal recipients and 12 of the top 26 scorers in competition represented countries other than the United States or Canada in the IMO. In 2007 five of the six Putnam Fellows were IMO Gold medalists and nine of the top 24 in the Putnam competition represented countries other than the United States or Canada in the IMO. In 2008 and 2009 four of the five Putnam Fellows were IMO Gold medalists. In 2008 five of the top 25 in the Putnam competition represented countries other than the United States or Canada in the IMO while in 2009 there were seven of the top 25.

Over the seventy competitions between 1938 and 2009 there have been only three perfect scores—one in 1987 and two in 1988. Although the top five scorers are always listed alphabetically, it is known that the 1987 perfect score was achieved by David Moews. What is amazing about this score is that the 1987 exam was a difficult one. The median score was one point and twenty-six points put one in the top two hundred (out of 2,170 participants). In 1987 the second highest score was 108, while the third highest score in 1988 was 119. The winners of the 1987 and 1988 competitions rank among

¹The MAA should create action figures for all the people who were Putnam Fellows three or more times.

²In the version of this article published in the *Monthly* I had Elkies as the youngest winner that I knew of.

the strongest groups of Putnam Fellows ever. Among them are Bjorn Poonen and Ravi Vakil, both four-time Putnam Fellows, David Moews and David Grabiner, both three-time Putnam Fellows, and Mike Reid, a two-time Putnam Fellow. In contrast to the 1988 scores, of the 1,260 contestants in the 1963 competition the highest score was sixty-two.

Two changes were made in 1992 regarding the recognition of individuals. In previous competitions the announcements of winners alphabetically identified the top ten as the five highest ranking participants and the next five highest. The next group of 30-35 highest ranking people was designated "Honorable Mention." In 1992 the announcement of the results put the top 25 into five categories: the five highest ranking individuals, the next five highest, the next five highest, the next ten highest. Beginning in 1997 the top 25 finishers were put into three categories: the five highest ranking individuals, the next ten highest, then the next ten highest. The number in the honorable mention group remained at about 30-35. The other change was the addition of an "Elizabeth Lowell Putnam Award" given from time to time to a female participant with a high score. Through 2009, there have been eight individual winners. Of these, Ioana Dumitriu and Alison Miller won it three times and Ana Caraiani and Melanie Wood won it twice. Dumitriu, Caraiani, and Wood were Putnam Fellows.

For most of the years between the late 1940s and the early 1990s Harvard far outpaced all other schools in the number of individuals receiving honorable mention status or higher. In 1991 Harvard had 11 and MIT had just 1 in that group. By 1993 MIT narrowed the margin to 8-6 in favor of Harvard. The first time that MIT surpassed Harvard was 1998 with the totals 11-9. In recognition of the significantly increasing number of participants, between 2002 and 2009 the number of those designed honorable mention has gradually increased from approximately 45 to 55. Since 1998 MIT has gradually increased its edge over Harvard from year to year in the number of individuals receiving honorable mention status or higher with the widest margin of 28-9 occurring in 2009. In fact MIT's total of 28 matches the total of the next three schools with the greatest number finishing honorable mention or higher—Caltech (11), Harvard (9), and Princeton (8). This deep pool of talent may have made it harder for MIT to beat Harvard in the team competition since between 1998 and 2009 Harvard has won the team competition six times to MIT's three times.

4. A PUTNAM WHO'S WHO. Over the years many distinguished mathematicians and scientists have participated in the Putnam. Among them are Fields Medalists John Milnor, David Mumford, Daniel Quillen, Paul Cohen, and John G. Thompson (Milnor, Mumford, and Quillen were Putnam Fellows; Cohen was in the second five; Thompson received Honorable Mention). Physics Nobel Laureates who have received Honorable Mention or better are Richard Feynman, a Putnam Fellow in 1939, Kenneth G. Wilson, a two-time Putnam Fellow, Steven Weinberg, and Murray Gell-Mann. The Nobel Prize winner in Economics John Nash (of "A Beautiful Mind" fame), to his great disappointment, finished in the second five of 147 individuals in 1947. Thompson won the Abel Prize in 2008. Eric Lander, one of the principal leaders in the Human Genome Project, finished in the second five in 1976. Both Mumford and Lander are MacArthur Fellows. Distinguished computer scientist Donald Knuth received Honorable Mention in 1959. American Mathematical Society Presidents who did well in the Putnam are Irving Kaplansky (Putnam Fellow, 1938), Andrew Gleason (Putnam Fellow, 1940, 1941, 1942), Felix Browder (Putnam Fellow, 1946), and AMS and MAA President Ron Graham (Honorable Mention, 1958). Putnam Fellows in National Academy of Sciences include (this list may not be exhaustive) Elwyn Berlekamp, Felix Browder, Eugenio Calabi, Andrew Gleason, Melvin Hochster, Roger Howe, Irving Kaplansky, George W. Mackey, John W. Milnor, David Mumford, Daniel G. Quillen, Lawrence A. Shepp, Peter W. Shor, and Kenneth G. Wilson. Many others who have done well in the Putnam have won the prestigious research awards given by the American Mathematical Society. The 1956 Harvard team had both a future Nobel prize winner (Wilson) and a future Fields medalist (Mumford). Both were Putnam Fellows that year and Harvard's team finished first.

One might wonder how the winners of the AMS/MAA/SIAM Morgan Prize for outstanding research by an undergraduate student have done in the Putnam Competition. Of the fifteen recipients through 2009 Wood, Barton, Kane, Manolescu, and Pixton have been Putnam Fellows.

5. CONCLUSION. Table 4 provides the top five scores and the median score for each competition between 1967 and 2009.³ Note that in five of those years the median score was zero and in six of them it was one! Between 1999 and 2009, only three times was the medium score greater than 1. Also observe that in 1995 only one point separated the highest and fifth highest scores. In the period 1967–2009 the largest gap between the top score and the fifth highest score was thirty-five, while the largest gap between highest top score and the second highest was twenty-two. The largest median in the period was 19; the average median score is 5.0; the median of the median scores is 3. The greatest number of zero scores occurred in 2006, when 2279 out of 3640 participants registered scores of zero. The highest percentage of scores of zero occurred in 2006 with 62.6% of the scores being zero. Table 5 gives the mean score, the percentage of the score of 0, and the score needed to finish in the top 500 in the period from 1987 to 2009.

Is there a lesson to be learned by examining the results of the Putnam competition? It seems that doing well on the Putnam exam correlates well with high achievement as a professional mathematician, but many of the best research mathematicians have not scored high on the Putnam and of course many have not even taken the exam.

Oh, by the way, the cadets of West Point beat Harvard that day in 1933. A cadet had the top individual score. Army's victory was reported in the newspapers and the Army team received a special letter of congratulations from the Army Chief of Staff, General Douglas MacArthur.

Reference [6], written by Putnam Fellows Kedlaya, Poonen, and Vakil, gives the problems with solutions and commentary from the Putnam competitions from 1985-2000. References [3] and [4] are articles that relate Putnam trivia. Reference [1] is an article that provides the views of the Putnam competition by a number of Putnam fellows. The web site <http://www.d.umn.edu/~jgallian/putnamfel/PF.html> provides information about Putnam Fellows.

³This was all the data that I could locate.

Table 1. Number of participants in the first sixty-nine competitions.

<i>Year</i>	<i>Number</i>	<i>Year</i>	<i>Number</i>	<i>Year</i>	<i>number</i>
1938	163	1963	1260	1986	2094
1939	200	1964	1439	1987	2170
1940	208	1965	1596	1988	2096
1941	146	1966	1526	1989	2392
1942	114	1967	1592	1990	2347
1946	67	1968	1398	1991	2325
1947	145	1969	1501	1992	2421
1948	120	1970	1445	1993	2356
1949	155	1971	1596	1994	2314
1950	223	1972	1681	1995	2468
1951	209	1973	2053	1996	2407
1952	295	1974	2159	1997	2510
1953	256	1975	2203	1998	2581
1954	231	1976	2131	1999	2900
1955	256	1977	2138	2000	2818
1956	291	1978	2019	2001	2954
1957	377	1979	2141	2002	3349
1958 S	430	1980	2043	2003	3615
1958 F	506	1981	2043	2004	3733
1959	633	1982	2024	2005	3545
1960	867	1983	2055	2006	3640
1961	1094	1984	2149	2007	3753
1962	1187	1985	2079	2008	3627
				2009	4036

Table 2. Number of teams 1975–2009.

<i>Year</i>	<i>Number</i>	<i>Year</i>	<i>Number</i>	<i>Year</i>	<i>Number</i>	<i>Year</i>	<i>Number</i>
1975	285	1984	264	1993	291	2002	376
1976	264	1985	264	1994	284	2003	401
1977	266	1986	270	1995	306	2004	411
1978	246	1987	277	1996	294	2005	395
1979	258	1988	257	1997	313	2006	402
1980	251	1989	288	1998	319	2007	413
1981	251	1990	289	1999	346	2008	405
1982	249	1991	291	2000	322	2009	439
1983	256	1992	284	2001	336		

Table 3. Winning teams in the first seventy competitions.

<i>Institution</i>	<i>First Place</i>	<i>Second Place</i>	<i>Third Place</i>	<i>Fourth Place</i>	<i>Fifth Place</i>	<i>Putnam Fellows</i>
Harvard University	27	10	12	5	1	98
California Inst. Technology	9	3	6	5	6	24
Massachusetts Inst. Technology	6	9	10	8	6	51
University of Toronto	4	5	4	4	1	23
Washington University	4	4		1	2	6
Duke University	3	2	6		1	6
Brooklyn College	3	1	1			5
Michigan State University	3			2		5
University of Waterloo	2	3	6	2	4	8
Cornell	2	3	1	1	2	5
Polytechnic Inst. Brooklyn	2	1				3
Princeton University	1	11	4	7	5	21
University of Chicago	1	3	3	1	3	10
U. California, Berkeley	1	1	2	4	2	16
U. California, Davis	1	1		1		2
Queen's University	1		1	1		1
Case Western Reserve	1			2	1	4
Yale University		3	1	4	3	9
Columbia University		2	3			8
Rice University		1	1	1	1	3
U. Pennsylvania		1	1	1		3
City College New York		1		4		10
Dartmouth		1			1	2
U. British Columbia		1			1	1
Oberlin College		1				
Carnegie Mellon			2	1		3
Cooper Union			2			1
U. California, Los Angeles			1		1	2
Harvey Mudd College			1		1	
U. Maryland, College Park			1		1	
New York University			1			3
Miami University			1			
Mississippi Women's College			1			
Stanford University				5	2	1

Table 3 (cont.). Winning teams in the first seventy competitions.

<i>Institution</i>	<i>First Place</i>	<i>Second Place</i>	<i>Third Place</i>	<i>Fourth Place</i>	<i>Fifth Place</i>	<i>Putnam Fellows</i>
U. Michigan, Ann Arbor				1	2	
Kenyon College				1		2
Swarthmore				1		1
University of Manitoba				1		1
Illinois Inst. Technology				1		
McGill University				1		1
University of Kansas					1	
U. of Minnesota Minneapolis						3
Purdue University						2
U. Alberta						2
U. California, Santa Barbara						2
U. Washington, Seattle						1

Table 4. Top five scores and median for the 1967–2009.

<i>Year</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>median</i>
1967	67	62	60	58	57	6
1968	93	92	89	85	85	10
1969	87	82	80	79	73	10
1970	116	107	104	97	96	4
1971	109	90	88	84	74	11
1972	85	79	66	63	59	4
1973	106	86	86	78	76	7
1974	77	70	62	61	57	6
1975	88	87	86	84	80	6
1976	74	70	68	64	61	2
1977	110	103	90	90	88	10
1978	90	77	74	73	71	11
1979	95	90	87	87	73	4
1980	73	72	69	68	66	3
1981	93	72	64	60	60	1
1982	98	90	88	85	82	2
1983	98	88	81	80	79	10
1984	111	89	81	80	80	10
1985	108	100	94	94	91	2
1986	90	89	86	82	81	19
1987	120	108	107	90	88	1
1988	120	120	119	112	110	16
1989	94	81	78	78	77	0
1990	93	92	87	77	77	2
1991	100	98	97	94	93	11
1992	105	100	95	95	92	2
1993	88	78	69	61	60	10
1994	102	101	99	88	87	3
1995	86	86	86	85	85	8
1996	98	89	80	80	76	3
1997	92	88	78	71	69	1
1998	108	106	103	100	98	10
1999	74	71	70	69	69	0
2000	96	93	92	92	90	0
2001	101	100	86	80	80	1
2002	116	108	106	96	96	3
2003	110	96	95	90	82	1
2004	109	101	99	89	89	0
2005	100	98	89	86	80	1
2006	101	99	98	92	92	0
2007	110	97	91	90	82	2
2008	117	110	108	102	101	1
2009	111	109	100	98	97	2

Table 5. Mean, percent 0, Top 500 cut off
1997–2009.

<i>Year</i>	<i>Mean</i>	<i>pct. 0</i>	<i>Top 500</i>
1997	7.3	47.7	12
1998	14.8	30.3	28
1999	6.3	60.2	11
2000	5.3	57.7	11
2001	8.9	44.9	20
2002	11.0	34.7	24
2003	7.1	27.8	18
2004	8.4	53.6	22
2005	7.9	46.7	20
2006	6.2	62.6	14
2007	7.0	42.5	21
2008	9.5	47.2	22
2009	9.5	43.7	22

6. APPENDIX I: EXAMINATION QUESTIONS FOR THE FIRST WILLIAM LOWELL PUTNAM MATHEMATICAL COMPETITION, APRIL 16, 1938.

MORNING SESSION: 9:00 to 12:00 NOON.

1. A solid is bounded by two bases in the horizontal planes $z = h/2$ and $z = -h/2$, and by such a surface that the area of every section in a horizontal plane is given by a formula of the sort $\text{Area} = a_0z^3 + a_1z^2 + a_2z + a_3$ (where as special cases some of the coefficients may be 0). Show that the volume is given by the formula $V = (1/6)h[B_1 + B_2 + 4M]$, where B_1 and B_2 are the areas of the bases, and M is the area of the middle horizontal section. Show that the formulas for the volume of a cone and a sphere can be included in this formula when $a_0 = 0$.

2. A can buoy is to be made of three pieces, namely, a cylinder and two equal cones, the altitude of each cone being equal to the altitude of the cylinder. For a given area of surface, what shape will have the greatest volume?

3. If a particle moves in a plane, we may express its coordinates x and y as functions of the time t . If $x = t^2 - t$ and $y = t^4 + t$, show that the curve has a point of inflection at $t = 0$, and that the velocity of the moving particle has a maximum at $t = 0$.

4. A lumberman wishes to cut down a tree whose trunk is cylindrical and whose material is uniform. He will cut a notch, the two sides of which will be planes intersecting at a dihedral angle θ along a horizontal line through the axis of the cylinder. If θ is given, show that the least volume of material is cut when the plane bisecting the dihedral angle is horizontal.

5. Evaluate the limits:

(a) $\lim_{n \rightarrow \infty} \frac{n^2}{e^n}$ (b) $\lim_{x \rightarrow 0} \frac{1}{x} \int_0^x (t + \sin 2t)^{1/t} dt$

6. A swimmer stands at one corner of a square swimming pool and wishes to reach the diagonally opposite corner. If w is his walking speed and s is his swimming speed ($s < w$), find his path for the shortest time. [Consider two cases: (a) $w/s < \sqrt{2}$ and (b) $w/s > \sqrt{2}$].

7. TAKE EITHER (a) or (b).

(a) Show that the gravitational attraction exerted by a thin homogeneous spherical shell at an external point is the same as if the material of the shell were concentrated at its center.

(b) Determine all the straight lines which lie upon the surface $z = xy$, and draw a figure to illustrate your result.

AFTERNOON SESSION: 2:00-5:00 P.M.

8. TAKE EITHER (a) or (b).

(a) Let A_{ik} be the cofactor of a_{ik} in the determine

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}.$$

Let D be the corresponding determinant with a_{ik} replaced by A_{ik} . Prove $D = d^3$.

(b) Let $P(y) = Ay^2 + By + C$ be a quadratic polynomial in y . If the roots of the quadratic equation $P(y) - y = 0$ are a and b ($a \neq b$), show that a and b are roots of the biquadratic equation $P[P(y)] - y = 0$. Hence write down a quadratic equation which will give the other two roots, c and d , of the biquadratic. Apply this result to solving the following biquadratic equation:

$$(y^2 - 3y + 2)^2 - 3(y^2 - 3y + 2) + 2 - y = 0.$$

9. Find all the solutions of the equation

$$yy'' - 2(y')^2 = 0$$

which pass through the point $x = 1, y = 1$.

10. A horizontal disc of diameter 3 inches is rotating at 4 revolutions per minute. A light is shining at a distant point in the plane of the disc. An insect is placed at the edge of the disc furthest from the light, facing the light. It at once starts crawling, and crawls so as always to face the light, at 1 inch per second. Set up the differential equation of motion, and find at what point the insect again reaches the edge of the disc.

11. Given the parabola $y^2 = 2mx$. What is the length of the shortest chord that is normal to the curve at one end?

12. From the center of a rectangular hyperbola a perpendicular is dropped upon a variable tangent. Find the locus of the foot of the perpendicular. Obtain the equation of the locus in polar coordinates, and sketch the curve.

13. Find the shortest distance between the plane $Ax + By + Cz + 1 = 0$ and the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$. (For brevity, let

$$h = 1/\sqrt{A^2 + B^2 + C^2} \text{ and } m = \sqrt{a^2A^2 + b^2B^2 + c^2C^2}.)$$

State algebraically the condition that the plane shall lie outside the ellipsoid.

7. APPENDIX II: POSSIBLE MOST DIFFICULT PROBLEMS ON PUTNAM COMPETITION BETWEEN 1974-2006

1979 competition (no positive scores)

A-6 Let $0 \leq p_i \leq 1$ for $i = 1, 2, \dots, n$. Show that

$$\sum_{i=1}^n \frac{1}{|x - p_i|} \leq 8n \left(1 + \frac{1}{3} + \frac{1}{5} + \dots + \frac{1}{2n-1} \right)$$

for some x satisfying $0 \leq x \leq 1$.

1999 competition (only one positive score—2 points)

B-4 Let f be a real function with a continuous third derivative such that $f(x), f'(x), f''(x), f'''(x)$ are positive for all x . Suppose that $f'''(x) \leq f(x)$ for all x . Show that $f'(x) < 2f(x)$ for all x .

1999 competition (only one positive score—2 points)

B-5 For an integer $n \geq 3$, let $\theta = 2\pi/n$. Evaluate the determinant of the $n \times n$ matrix $I + A$, where I is the $n \times n$ identity matrix and $A = (a_{jk})$ has entries $a_{jk} = \cos(j\theta + k\theta)$ for all j, k .

Acknowledgment. The data in Table 3 was kindly provided by Jerry Heuer, Leonard Klosinski, and Jerry Alexanderson. I wish to thank Doug Jungreis, Kiran Kedlaya, Bjorn Poonen and Ravi Vakil for their comments on a draft of the article that appeared in the Monthly. No doubt this article set a record for the most number of Putnam Fellows to read a draft of a Monthly article.

References

- [1] G. L. Alexanderson, How Putnam fellows view the competition, Focus December, 2004 *Focus* 14-15.
- [2] D. C. Arney, Army beats Harvard in football and mathematics!, (September, 1994) *Math Horizons* 14-17.
- [3] J. A. Gallian, Fifty years of Putnam trivia, *Amer. Math. Monthly* **96** (1989) 711-713.
- [4] J. A. Gallian, Putnam trivia for the 90s, *Amer. Math. Monthly* **107** (2000) 733-735.
- [5] J. A. Gallian, The First sixty-six years of the Putnam competition, *Amer. Math. Monthly* **111** (2004) 691-699.
- [6] K. S. Kedlaya, B. Poonen, and R. Vakil, *The William Lowell Putnam Mathematical Competition 1985-2000: Problems, Solutions, and Commentary*, Mathematical Association of America, Washington, D.C., 2002.
- [7] G. W. Mackey, The William Lowell Putnam Mathematical Competition *Amer. Math. Monthly* **55** (1949) 630-632.

Joe Gallian received a B.A. degree from Slippery Rock State University in 1966 and Ph.D. from Notre Dame in 1971. He has been at the University of Minnesota Duluth since 1972, where he is a University Distinguished Professor of Teaching. He serves on the editorial board of this MONTHLY and *Math Horizons* and served one term on the editorial board of the *Mathematics Magazine*. He has received the MAA's Haimo Award for Distinguished Teaching, and the MAA's Allendoerfer and Evans awards for exposition. He is a codirector of Project NExT and has been an MAA Polya Lecturer and a Second Vice President of the MAA. In 2003 he was named Minnesota Professor of the Year by the Carnegie Foundation for the Advancement of Teaching. Since 1977 he has run a summer undergraduate research program that has had a total of 168 participants through 2009. He has had the good fortune to work with thirty Putnam Fellows in his summer programs.

Department of Mathematics and Statistics, University of Minnesota Duluth, Duluth, MN 55812
jgallian@d.umn.edu